

Inflation, Quarterly Financial Statements and Fraud: Benford's Law and the Brazilian Case.

Autoria: Alexandre Majola Gava, Luiz Roque de Souza Vitiello Jr.

Abstract

There is an increasing concern in the financial markets regarding the quality of financial statements issued by public companies. Actually, investors are mainly concerned about what they cannot see; that is the possibility of them being ripped off. Although fraud is usually generated inside the company there are exogenous factors that may contribute to the companies' decision of reporting a fraudulent financial statement. We apply Benford's law to approximately two thousand quarterly balance sheets of 44 Brazilian public companies from 1986 to 2002 in order to test the probability of occurrence of fraud. Our findings suggest an inverse relationship between inflation and fraud: the higher the former the lower the latter. The results also suggest that the probability of occurrence of fraud is higher in the financial statements published in the first three quarters than in the one published in the fourth quarter.

1. Introduction

There is an increasing concern in the financial markets regarding the quality of financial statements issued by public companies. This concern is not only related to how profitable the companies are or to their earning per share. Actually, investors are mainly concerned about what they cannot see; that is the possibility of them being ripped off. But as corporations become larger their financial statements become more complex and as a consequence, fraud detection becomes more difficult and also more expensive. In addition to that audits usually focus only on areas more exposed to risk, and this may also contribute to fraud.

Although fraud is usually generated inside the company there are exogenous factors that may contribute to the companies' decision of reporting a fraudulent financial statement. One could consider the decision of reporting a fraudulent financial statement as a simple trade off between the cost associated of being caught and the cost of exposing the true financial situation of the company. The smaller the former the higher the probability of fraud. Then, weak regulation, lack of appropriate punishment and corruption in the government are examples of systematic factors that may reduce both the cost and the likelihood of being caught.

In this paper we try to investigate empirically whether changes on inflation levels have an impact on the level of fraud. In order to do so, we use quarterly financial statements of Brazilian companies from 1986 to 2002. It is relevant to note that the inflation rate in Brazil decreased enormously after mid 1994, due a successful set of economical reforms that took place earlier that year. This allows us to work with two completely different periods: the first one where there was high inflation (before 1994) and the second one with low inflation (after 1994).

In order to capture the possibility of occurrence of fraud we use Benford's law,ⁱ which has successfully been applied to fraud analyses and can easily cope with large datasets. This law establishes that in certain data sets the frequency of occurrence of the digit n , for $n=1, 2, \dots, 9$, is greater for some digits than for others, which means that the distribution of n is not uniform, $f(n) \neq (1/9)$. Therefore if one finds that the observed frequency of occurrence of the first digit is different to its expected frequency according to Benford's law,ⁱⁱ there is a probability of fraud.

Thus, from the assumption that Benford's law can properly forecast the frequency of occurrence of the first digit on financial statements of Brazilian companies, we apply it to 1946 quarterly balance sheets of 44 Brazilian companies during 1986 to 2002 period. We found a far from intuitive relation between inflation and fraud: the probability of fraud seems to be lower in high inflation periods and vice versa. We also found that there is a statistical difference when testing the quarterly statements separately in a low inflation scenario. The results suggest that in such scenario the probability of occurrence of fraud is higher in the financial statements published in the first three quarters than in the one published in the fourth quarter.

This paper is divided as follows: Benford's law is discussed in the following section; the data and the methodology applied are presented in the third section; in section four the results are presented and in section five they are analysed and linked to Brazilian economical situation; section six concludes.

2. The Benford' Law

The Benford's Law states that the frequency of occurrence of some digits is greater for some digits than for others. This phenomenon was initially observed by Newcomb (1881) who noticed that the first pages of logarithmic tables would wear out faster than others, which suggests that the ten digits do not occur with equal frequency. Newcomb concluded that "The law of probability of the occurrence of numbers is such that all mantissae of their logarithms are equally probable" (p. 40)

Half a century later Benford (1938) calculated the frequency of occurrence of the first digit of multi-digit numbers obtained from a large and diversified database. Benford concluded that the digit 1 occurs more frequently than the digit 2; the digit 2 occurs more frequently than the digit 3 and so on. Based on this result, Benford suggested that the observed frequency of occurrence could be approximated using the following equation

Equation 1
$$F(n) = \log_{10} \left(\frac{n+1}{n} \right), \text{ for } n=1,2, \dots, 9$$

With time, this phenomenon became known as Benford's Law. Although neither Newcomb nor Benford provided a formal mathematical proof for this phenomenon, there have been several attempts to explain it. Pinkham (1961), Raimi (1969, 1976) and Hill (1995) among others provide a good revision on this subject.

It is likely that Benford (1938) himself was the first to conduct an empirical test on the Benford's Law. He used a large and diversified database containing information about population, atmospheric pressure, mortality rate and mathematical constants among others, with a total of 20,229 observations.

Since then, the Benford's Law has been applied to different topics, such as physical constants by Burke and Kincanon (1971), radioactivity by Buck, Merchant and Perez (1992), population size and density by Sandrom and Hayford (2002), economics by Varian (1972), stock index return by Ley (1996) and to income tax data by Nigrini (1996).

It has also been successfully applied to the detection of fraud in financial statements with practical and instant results. This is because the analysis of financial statements using the Benford's Law allows a quick identification of balance sheet accounts that could have been manipulated. The test is done by checking the frequency of occurrence of the first digit of

each one of the balance sheet accounts against the frequency predicted by Benford's Law (i.e. comparing the observed frequency to the expected frequency). The cases where observed and expected frequencies do not match may indicate the existence of fraud. For instance, if the frequency of appearance of balance sheet accounts starting with the number 5 does contradict the frequency suggested by Benford's Law, then all the balance sheet accounts starting with the number 5 should be included in the sample for audit.

Moore and Benjamin (2004) applied the Benford's Law to a case study of a small chemical plant in the USA, which helped them to find fraud in the company. Negrini (1999), Santos, Diniz and Corrar (2005) and Quick and Wolz (2005) also present several cases where the Benford's Law was successfully applied in the detection of inconsistency and fraud.

3. Data and Methodology

The sample consisted of 1946 unconsolidated quarterly balance sheets of 44 public Brazilian companies traded on BOVESPA, the largest stock exchange in Latin America. The data were collected from Economatica database, covering the period from 1986 to 2002.

From 1986 to 1994 inflation rates in Brazil were extremely high as shown in Table 1. In order to eliminate this effect from the balance sheets analysed they were adjusted by the IGP-DI.ⁱⁱⁱ Although Benford's law does not require any adjustments, removing the inflation effect from the financial statements is a standard procedure among analysts in Brazil since nominal values are meaningless given such a high levels of inflation.

Table 1: Inflation in Brazil (1986-1994)

Year	IGP-DI
1986	65,0%
1987	415,9%
1988	1037,5%
1989	1782,9%
1990	1476,7%
1991	480,2%
1992	1157,8%
1993	2708,4%
1994	909,7%

Table 2: Inflation in Brazil (1995-2003)

Year	IGP-DI
1995	14,8%
1996	9,3%
1997	7,5%
1998	1,7%
1999	20,0%
2000	9,8%
2001	10,4%
2002	26,4%

In the second semester of 1994 the Brazilian government successfully introduced a set of economic reform that slowed the inflationary process, as shown in Table 2. Among these

changes were the introduction of a new currency in July 1994 and the abolishment of inflationary adjustments^{iv} to financial statements in 1995.

Therefore, one could say that the Brazilian macroeconomic scenario during the last couple of decades offers a unique setting, which rises an interesting question: Should one expect high (low) probability of fraud during periods of high (low) inflation? In order to address this question we isolate the impact of inflation in our sample by using three distinct periods: the whole period (1986–2002), the period of high inflation (1986–1993), and the period of low inflation (1995–2002).

In the following analysis the tests are run as follows: (i) For each quarter, we selected the first digit of each one of the balance sheet accounts of all companies; (ii) We calculated the observed frequency, i.e. we counted the number of times each digit appeared and divided the result by the number of times all digits appeared; (iii) We compared the observed frequency to the expected frequency calculated according to Equation 1. If for a certain digit n , observed and expected frequencies statistically similar, we assumed that the financial statements conform to Benford's Law and therefore it is unlikely that there were fraud in the balance sheet accounts that start with the digit n .

In order to test the significance of the results we applied the t and the z statistical tests. The z test is given by the following equation

Equation 2
$$z_i = (P_i - p_i) (p_i \times q / N_i)^{-1/2},$$

where P_i is the observed frequency of digit i , p_i is the expected frequency of digit i , $q = 1 - p$, and N is the number of observations of digit i . The intuition behind the test is that the closer observed and expected frequencies are the smaller the numerator, which means a small z and as a consequence, a small probability of fraud.

To check how well the observed frequencies fit to the expected frequencies for the whole digits altogether, we apply the chi-squared test, given by

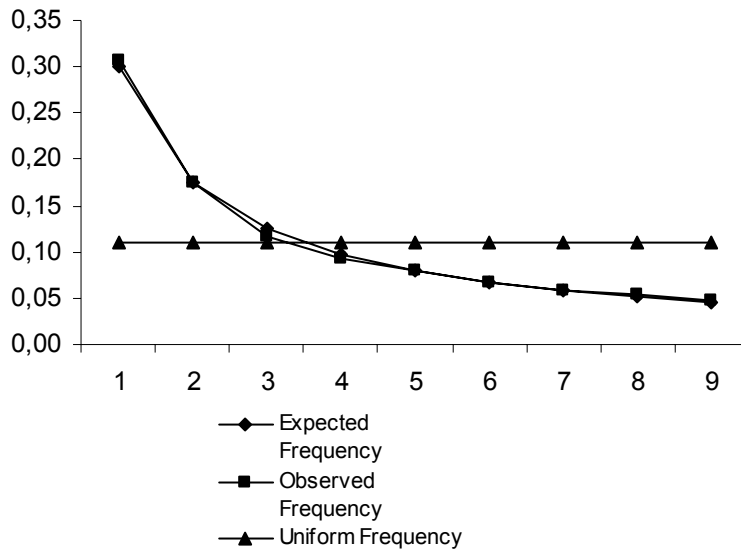
Equation 3
$$\chi^2 = \sum_{j=1}^N \frac{(n.o_j - n.e_j)^2}{n.e_j}$$

where o_j is the j^{th} observed frequency, e_j is the j^{th} expected frequency, n_j is the number of observations in j^{th} frequency, and N is the number of frequencies. As before, the intuition is that the closer o_j is to e_j the smaller the value of χ^2 , suggesting a small probability of fraud for the digits altogether.

4. The results

Considering that it would be fair to assume that the observed frequency might be uniformly distributed,^v we present in Figure 1 the observed frequency, the expected frequency calculated from Equation 1 and the uniform density. It is possible to see that while the observed frequency seems to fit well to the expected frequency, it definitely does not bear any relation with the uniform density. Therefore we rule out the uniform density and focus exclusively on the application of Benford's Law.

Figure 1: Observed, expected and uniform frequencies (1986-2002)



We then group the balance sheets of each company, and test whether the observed and expected frequencies are the same for each digit at a level of significance of 5%. Table 3 presents the results where on average approximately three quarters of the companies' aggregate balance sheet do fit to Benford's law. Taking digit 9 as an example, Table 3 shows that the balance sheets of 37 companies out of 44 have an observed frequency for the first digit that follows the expected frequency according to Benford's law. This result highlights the general idea that Benford's law represents reasonably well the frequency of occurrence of the first digit of the sample being analysed.

Table 3: Significant digit by company

Digit	Company	%
1	29	65,9%
2	34	77,3%
3	39	88,6%
4	38	86,4%
5	35	79,5%
6	36	81,8%
7	36	81,8%
8	35	79,5%
9	37	84,1%

Note: (i) Company: the number of companies that conform to Benford's law (ii) % = Company/44.

a) The inflation effect

We now start to address the impact of inflation on fraud. First, using the same method as in Table 3, we grouped the financial statements of all companies together and repeated the test again. The values for the observed and expected frequencies and the results for the statistical

tests are presented in Table 4. According to the z test, the observed frequency of digits 1, 3, 4 and 8 do not conform to the frequency predicted by Benford's Law. Based on the chi-squared test the goodness of fit of the observed frequency to the expected frequency for all digits is not significant.

Table 4: Results for the whole sample 1986-2002

Digit	Expected Frequency	Observed Frequency	Number of Observations	z	Chi-Squared
1	30,10%	30,74%	22.074	3,76	9,90
2	17,60%	17,60%	12.636	(0,01)	0,00
3	12,50%	11,63%	8.349	(7,07)	43,65
4	9,70%	9,37%	6.729	(2,98)	7,96
5	7,90%	7,93%	5.696	0,32	0,10
6	6,70%	6,77%	4.862	0,76	0,55
7	5,80%	5,92%	4.249	1,34	1,72
8	5,10%	5,33%	3.826	2,78	7,37
9	4,60%	4,70%	3.378	1,33	1,71
Total			71.799		72,96

Note: (i) Expected frequency is calculated according to Equation 1; (ii) Observed frequency is computed directly from the sample; (iii) z is calculated according to Equation 2; (iv) Chi-squared is calculated according to Equation 3.

We then repeat the same test for the high and low inflation periods. The results for each set are presented in Table 5 and Table 6 respectively. For the high-inflation period (Table 5) only the result for digit 5 is statistically significant.^{vi} That is, digit 5 does not 'follow' Benford's Law.

For the low-inflation period (Table 6), the results are similar for the test using the whole sample (Table 4) – the observed frequency for digits 1, 3, 4 and 8 is not statistically similar to the expected frequency according to Benford's Law.

The chi-squared test suggests that there is no significant difference between the observed and expected frequencies for the high inflation period only, which seems to contradict intuition since one could expect that high levels of inflation would distort the figures in the balance sheet and consequently reduce its transparency.

Table 5: Results for the high-inflation period (1986-1993)

Digit	Expected Frequency	Observed Frequency	Number of Observations	z	Chi-Squared
1	30,10%	30,68%	6.326	1,82	2,34
2	17,60%	17,63%	3.634	0,09	0,01
3	12,50%	12,43%	2.562	(0,33)	0,09
4	9,70%	9,65%	1.990	(0,24)	0,05
5	7,90%	7,46%	1.537	(2,38)	5,16
6	6,70%	6,53%	1.346	(1,00)	0,90
7	5,80%	6,12%	1.262	1,96	3,67
8	5,10%	4,94%	1.019	(1,04)	1,00
9	4,60%	4,56%	940	(0,29)	0,07
Total			20.616		13,29

Note: (i) Expected frequency is calculated according to Equation 1; (ii) Observed frequency is computed directly from the sample; (iii) z is calculated according to Equation 2; (iv) Chi-squared is calculated according to Equation 3

Table 6: Results for the low-inflation period (1994-2002)

Digit	Expected Frequency	Observed Frequency	Number of Observations	z	Chi-Squared
1	30,10%	30,75%	15.980	3,21	7,22
2	17,60%	17,56%	9.125	(0,26)	0,05
3	12,50%	11,32%	5.881	(8,17)	58,34
4	9,70%	9,30%	4.832	(3,11)	8,70
5	7,90%	8,10%	4.212	1,72	2,74
6	6,70%	6,87%	3.570	1,53	2,21
7	5,80%	5,86%	3.045	0,56	0,31
8	5,10%	5,49%	2.852	4,01	15,30
9	4,60%	4,76%	2.476	1,77	3,04
Total			51.973		97,91

Note: (i) Expected frequency is calculated according to Equation 1; (ii) Observed frequency is computed directly from the sample; (iii) z is calculated according to Equation 2; (iv) Chi-squared is calculated according to Equation 3

b) The quarter effect

In this section we split the data by quarters and then we repeat the test carried out in previous section. One could expect that there would not exist any significant difference between the quarters. Although this is true for the first three quarters, it is not for the fourth quarter. For this reason, considering that the results for the first three quarters are similar we present their aggregate value only.

The results for the fourth quarter for the whole period are presented in Table 7. Table 8 and Table 9 show the results for the high inflation period and the low inflation period respectively. There are two digits that do not conform to Benford's law in Table 7 and Table 9 and only one that does not conform in Table 8.

Table 7: Results for the fourth quarter (1986-2002)

Digit	Expected Frequency	Observed Frequency	Number of Observations	z	Chi-Squared
1	30,10%	31,39%	6.107	3,91	10,74
2	17,60%	17,61%	3.426	0,02	-
3	12,50%	11,50%	2.237	(4,24)	15,64
4	9,70%	9,36%	1.822	(1,59)	2,25
5	7,90%	7,74%	1.506	(0,84)	0,63
6	6,70%	6,65%	1.293	(0,32)	0,09
7	5,80%	5,75%	1.118	(0,34)	0,10
8	5,10%	5,30%	1.031	1,25	1,51
9	4,60%	4,71%	916	0,70	0,49
Total			19.456		31,45

Note: (i) Expected frequency is calculated according to Equation 1; (ii) Observed frequency is computed directly from the sample; (iii) z is calculated according to Equation 2; (iv) Chi-squared is calculated according to Equation 3

Table 8: Results for the fourth quarter (1986-1993)

Digit	Expected Frequency	Observed Frequency	Number of Observations	z	Chi-Squared
1	30,10%	31,73%	2.242	2,98	6,27
2	17,60%	17,20%	1.215	(0,90)	0,65
3	12,50%	12,70%	897	0,48	0,22
4	9,70%	9,09%	642	(1,76)	2,74
5	7,90%	7,40%	523	(1,57)	2,21
6	6,70%	6,54%	462	(0,56)	0,27
7	5,80%	5,55%	392	(0,93)	0,77
8	5,10%	5,05%	357	(0,21)	0,03
9	4,60%	4,74%	335	0,54	0,31
Total			7.065		13,47

Note: (i) Expected frequency is calculated according to Equation 1; (ii) Observed frequency is computed directly from the sample; (iii) z is calculated according to Equation 2; (iv) Chi-squared is calculated according to Equation 3

Table 9: Results for the fourth quarter (1994-2002)

Digit	Expected Frequency	Observed Frequency	Number of Observations	z	Chi-Squared
1	30,10%	31,19%	3.865	2,64	4,91
2	17,60%	17,84%	2.211	0,70	0,42
3	12,50%	10,81%	1.340	(5,69)	28,17
4	9,70%	9,52%	1.180	(0,68)	0,40
5	7,90%	7,93%	983	0,12	0,02
6	6,70%	6,71%	831	0,01	-
7	5,80%	5,86%	726	0,26	0,07
8	5,10%	5,44%	674	1,70	2,80
9	4,60%	4,69%	581	0,45	0,21
Total			12.391		37,00

Note: (i) Expected frequency is calculated according to Equation 1; (ii) Observed frequency is computed directly from the sample; (iii) z is calculated according to Equation 2; (iv) Chi-squared is calculated according to Equation 3

The aggregate results for the first three quarters for the whole period are presented in Table 10.

Table 11 presents the results for the period of high inflation and Table 12 shows the results for the period of low inflation. There are six digits that statistically do not conform to Benford's law in Table 10 and Table 12, and only two in Table 11.

Table 10: Results for the first, the second and the third quarters (1986-2002)

Digit	Expected Frequency	Observed Frequency	Number of Observations	z	Chi-Squared
1	30,10%	30,54%	16.140	2,22	3,45
2	17,60%	17,26%	9.122	(2,04)	3,42
3	12,50%	11,97%	6.323	(3,72)	12,07
4	9,70%	9,26%	4.894	(3,41)	10,48
5	7,90%	7,84%	4.141	(0,55)	0,27
6	6,70%	7,10%	3.751	3,65	12,52
7	5,80%	5,83%	3.082	0,31	0,10
8	5,10%	5,22%	2.757	1,22	1,43
9	4,60%	4,98%	2.633	4,19	16,82
Total			52.843		60,56

Note: (i) Expected frequency is calculated according to Equation 1; (ii) Observed frequency is computed directly from the sample; (iii) z is calculated according to Equation 2; (iv) Chi-squared is calculated according to Equation 3

Table 11: Results for the first, the second and the third quarters (1986-1993)

Digit	Expected Frequency	Observed Frequency	Number of Observations	z	Chi-Squared
1	30,10%	30,25%	4.139	0,37	0,10
2	17,60%	17,52%	2.398	(0,24)	0,04
3	12,50%	12,60%	1.724	0,34	0,11
4	9,70%	9,64%	1.319	(0,26)	0,05
5	7,90%	7,40%	1.013	(2,17)	4,28
6	6,70%	6,87%	940	0,78	0,59
7	5,80%	6,23%	852	2,11	4,29
8	5,10%	4,79%	655	(1,69)	2,64
9	4,60%	4,71%	644	0,57	0,34
Total			13.684		12,44

Note: (i) Expected frequency is calculated according to Equation 1; (ii) Observed frequency is computed directly from the sample; (iii) z is calculated according to Equation 2; (iv) Chi-squared is calculated according to Equation 3

Table 12: Results for the first, the second and the third quarters (1994-2002)

Digit	Expected Frequency	Observed Frequency	Number of Observations	z	Chi-Squared
1	30,10%	30,65%	12.001	2,35	3,89
2	17,60%	17,17%	6.724	(2,24)	4,09
3	12,50%	11,74%	4.599	(4,53)	17,88
4	9,70%	9,13%	3.575	(3,82)	13,14
5	7,90%	7,99%	3.128	0,64	0,38
6	6,70%	7,18%	2.811	3,78	13,38
7	5,80%	5,69%	2.230	(0,90)	0,75
8	5,10%	5,37%	2.102	2,40	5,51
9	4,60%	5,08%	1.989	4,52	19,56
Total			39.159		78,58

Note: (i) Expected frequency is calculated according to Equation 1; (ii) Observed frequency is computed directly from the sample; (iii) z is calculated according to Equation 2; (iv) Chi-squared is calculated according to Equation 3

Considering the results for the low inflation only, there are two digits that do not conform to Benford's law in the fourth quarter and six in the other quarters. For the period of high inflation, however the results are similar. The results of the chi-squared test suggest that both samples fit similarly to Benford's law. In particular, as in the previous section, the chi-squared test show that the Benford's law fits particularly well in the high inflation scenario, suggesting a low occurrence of frauds.

5. Analysis of the results

In this section we analyse the results obtained and attempt to relate them to the previous literature, in particular to the differences between the economical situation in the periods of low and high inflation.

The results so far suggest that there is a higher probability of occurrence of fraud during low inflation periods than during high inflation periods. They also suggest that fraud is more likely to happen in the first three quarters than in the fourth quarter. In particular, one can say that: (i) the probability of fraud is small in a period of high inflation; (ii) it is also small for the fourth quarter balance sheets and finally; (iii) it is high for first, second and third quarters balance sheets published in a period of low inflation.

The results concerning inflation are quite counterintuitive since it suggests that an environment with high inflation seems to have a lower probability of fraud than one with low inflation. Although there is not a clear answer for this puzzle there were some factors that in may help to justify the puzzle: (i) soon after the same economical reforms that reduced the level of inflation there were other deep changes in the Brazilian economy. In particular, before 1993 most of the more liquid stocks traded in the Brazilian Stock Exchange were monopolies owned by the government. These included the steel sector, the telephonic sector^{vii}, and the energy sector^{viii} among others. Since these companies were considered strategic by the government, one could say that their financial results were not the government's primary concern, which suggests that their managers did not have any incentives for committing fraud; (ii) companies traded on the Brazilian Stock Exchange strongly increased their debt level after 1994 (see Gava and Vieira (2005)). Since debt issue is somehow related to good results, one could say that there was an additional incentive for fraud, which leads to an increase on the probability of it happening and finally; (iii) Brazilian companies issued ADR in the North-American market, leading to an additional pressure for good results.

In terms of the "fourth quarter effect", it is possible that companies may try to postpone taxes as much as possible by fraudulently changing the figures in the financial statements in order to reduce the profits in the first three quarters.^{ix} If this is the case, the small profit in the first three quarters would be completely compensated in fourth quarter when the company would show the full (and presumably correct) figure for profits. One could argue that if companies were willing to commit fraud to pay less tax there is no reason to believe that they would show the right profit figures in the fourth quarter.

Although this is true, it is important to note that bad results in the fourth quarter may have a stronger negative impact on the companies' credit rating as well as on investors' confidence. The fourth quarter is the end of the fiscal year, and so investors tend to focus more on it than on other quarters.

6. Concluding remarks

In this paper we applied Benford's law to 1946 financial statements of 44 Brazilian companies from 1986 to 2002 in order to test the probability of occurrence of fraud. Our findings suggest that a high inflation scenario is strongly related to a reduction on the probability of fraud. They also suggest that there is a low chance of fraud in the fourth quarter financial results. Finally, we found that the occurrence of fraud is more likely to happen in the first, second and third quarters balance sheets in a low inflation scenario. These results are strongly related to the Brazilian economical situation at that time.

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ⁱ Alternatively we could use the actual number of frauds reported during the period studied but this would give us information only about the reported frauds.

ⁱⁱ It is important to note that although we focus on the first digit only, the law can also be applied to the nth digit of any multi-digit number as claimed by Benford (1938).

ⁱⁱⁱ The IGP-DI (General Price Index – Internal Supply) was created in 1944 and is calculated by Fundação Getúlio Vargas, a non-governmental institution.

^{iv} The Brazilian law required that some balance sheet accounts (e.g. investments and fixed assets) should be adjusted to inflation while others (e.g. cash and receivables) should not.

^v That is, that all digits would have a probability of occurrence of 1/9.

^{vi} Note that digits 1 and 7 are statistically different at a level of 10%.

^{vii} The traded volume of the former Telebras, for instance, was responsible for about 40% of the total volume of the IBOVESPA.

^{viii} This includes production, transmission and distribution.

^{ix} Note that losses can be compensated up to a certain limit in the following quarter.