Robust Asset Allocation in Emerging Stock Markets

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Abstract

Financial data are heavy tailed containing extreme observations. We use a robust covariance estimator to define the center and orientation of the data and also provide an illustration of the usefulness of the proposed procedure to efficiently allocate funds among emerging stock markets. Several aspects of the out-of-sample performance of the robust and classical portfolios were investigated. The resulting robust portfolios can yield higher cumulative returns. For any given type of portfolio in the efficient frontier, the robust portfolios showed a more concentrated distribution with higher expected returns. The robust estimates were able to reduce instability of the optimization process. Finally, we found that this stability property carried over to the weights associated with the robust portfolios. We recommend that a robust covariance matrix be used to solve emerging stock markets allocation problems and believe that our technique enjoys a key advantage. Employing a simple change in the covariance matrix, we can use any commercially available optimizer to obtain robust portfolio weights.

I. Introduction

In this paper we demonstrate a technique that computes a covariance matrix contaminated with extreme values information to make it represent both most usual days and extreme value days. We call classical sample estimates the average returns and the covariance matrix of each historical series of returns studied in an asset allocation problem. Classical sample estimates of covariance matrices for Markowitz (1959) mean-variance optimization can be severely distorted by extreme values. This problem can be particularly serious in more volatile emerging stock markets. Current research deals with the fat-tails problem of emerging markets using the safety-first concept and extreme value theory, such as in Haque et al (2004) and Susmel (2001). These authors find that emerging market weights in global portfolios should be between 10% and 15%.

An alternate strategy to deal with this problem is to make either the optimizer robust or the input robust. In this paper we call a robust portfolio one that uses robust covariance estimates. Robustness means that these estimates have been treated in a way that the effect of extreme values is minimized. We will explain how we accomplish this further in the article. Robust portfolio optimization use techniques that assume that the data is not robust to extreme values instead of using robust estimates. These optimization techniques are complex and, to the best of our knowledge, are not easily commercially available. Our concern in this paper is to present examples of a technique that generates a robust covariance matrix that can be use as input to any commercially non-robust portfolio optimizer.

We offer a method to contaminate the covariance matrix with most extreme values observed in the sample, yet retaining the desired distribution properties so that the data represent most business days. Thus, one could simply use standard commercially available optimizers with a robust covariance matrix. Our technique was developed with a long-term asset allocation application in mind. Hence our concern is to keep the characteristics of most business days in the data without ignoring that extreme values represent part of the usual emerging stock market behavior.

Traditional finance models assume the multivariate normal distribution for a portfolio of independent and identically distributed (iid) assets. Such is the case of conventional uses of the Mean-Variance (MV) model of Markowitz (1959). Based on this assumption, the resulting
procedure simply requires estimates of the sample mean and sample covariance. They are the maximum likelihood estimators and possess the desirable statistical properties. However, their asymptotic breakdown point is equal to zero (Maronna, 1976), which means that they are adversely affected by extreme observations, which render the data meaningless. The concept of breakdown point is related to the amount of extreme values which can “break down” the estimator. It is a measure that answers the question “what is the maximum fraction of atypical values in the sample with which the estimator still gives reliable information?” For example, the breakdown point of the sample mean is zero, the smallest possible value, reflecting its high sensitivity to extreme values.

Let us suppose that the joint distribution of two time series could be represented by an elliptic distribution. The effects of atypical points on the ellipsoid associated with an estimate of the covariance structure of these two time series are at least two: (1) they may inflate its volume and (2) they may tilt its orientation (Johnson and Wichern, 1990). The first effect is related to inflated scale estimates. The second is the more serious and may show up by switching the signs of correlations. One can easily imagine that using such distorted estimates in an asset allocation problem would provide the user with inappropriate weights. We offer an emerging market illustration of our technique that permits correcting for the tilt in the covariance matrix orientation, without dropping most of the extreme observations.

Extreme observations may or may not be considered outliers (this is a polemic discussion topic), but they certainly seem to be related to a data generating process different from the one yielding the vast majority of observations. These atypical observations constitute a small proportion of the data set. In this paper we will use the terms “outliers” to denote “extreme values” or “atypical observations”, regardless of whether they are actually outliers or not in a more strict sense. Examples of treating atypical observations with extreme value models are Haque et al (2004), Susmel (2001), Hartman et al (2001) and Embrechts et al (1997). An example of a treatment with regime-switching models is Ang and Bekaert (2002). In this paper we use a variation of the well known high breakdown point Minimum Covariance Determinant (MCD) estimator to obtain robust efficient frontiers to construct robust portfolios in emerging markets.

We compare the performances of the robust and classical MV optimal portfolios and demonstrate that the robust portfolios may yield higher cumulative returns besides possessing more stable weight structures. The remainder of this paper is organized as follows. In Section II we propose a robust estimation procedure for the inputs of the MV model. To illustrate, we use emerging markets data in Section III. In Section IV we summarize our results.

II. Robust Inputs for MV Optimization

The MV Model is probably the most used in practical asset allocation applications. Estimation of the efficient frontier is almost always done via the sample mean $\bar{x}$ and sample covariance matrix $\mathbf{S}$. However, different statistical estimates define different efficient frontiers. One of the most important limitations of MV optimization in practice is the lack of optimality presented by these classical estimates.

To obtain a good representation for the p-dimensional data we propose to estimate the covariance matrix using
\[(1 - \epsilon)\mathbf{S}_1 + \epsilon\mathbf{S}_2\] (1)
where $\epsilon$ is defined as a contaminating proportion.

The $p \times p$ covariance matrix $\mathbf{S}_1$ represents the (predominant) dependence structure of the usual business days, or, in other words, the covariance structure of the data cloud without the outliers. $\mathbf{S}_2$ is the covariance matrix of an extended data cloud containing also most of the atypical observations.
In equation 1, the ellipsoids associated with \(1\) and \(2\), for a fixed sample matrix \(X\), have the same orientation but different volumes. These characteristics are derived from the choice of the same eigenvectors for \(1\) and \(2\). In practice, because \(\varepsilon\) is small, the contaminating distribution in equation 1 typically produces spurious extreme observations seeming to follow an orientation structure different from that observed during normal days, or \(1\). These are the observations occurring during stress periods when we may observe different (greater) correlations. When using the classical sample covariance matrix \(S\), these few points can tilt the orientation of the axes of its associated ellipsoid.

To avoid this problem, we propose to estimate the correct orientation of the data using the high breakdown point Minimum Covariance Determinant (MCD) estimate (Rousseeuw, 1985). The volumes of \(1\) and \(2\) are estimated based, respectively, on the volumes of the robust MCD and classical \(S\) covariance estimates. We estimate the proportion \(\rho\) empirically.

The MCD is a covariance affine equivariant estimator which attains the maximum possible breakdown point (approximately 0.5). For a given integer \(h\), the MCD location estimator \(\hat{\mu}_h(X)\) is defined as the mean of the \(h\) points of \(X = (x_1, x_2, \ldots, x_n)\) for which the determinant of the sample covariance is minimal.

Let us denote by \(h^+\) the number of points used to obtain the MCD. Thus, \(n - h^+\) extreme points were not used to compute the covariance estimate and this information is used to empirically compute the proportion \(\rho\). We can interpret the MCD estimator as able to measure the “outlyingness” of any data point relatively to the center of the collection.

In summary, we assume that a set of stock returns \(X \in \mathbb{R}^n\) possesses a distribution that is a mixture of two elliptical distributions with same center and covariance matrices intended to represent the usual days and most atypical days. This fraction of observations results from days when larger volatility is observed, with no change in the strength and sign of correlations. Contamination here does not mean errors, and is merely a mechanism to model the data. We denote the robust estimators of the location and covariance matrix of the data by \((\hat{\mu}_{\text{emp}}, \hat{\Sigma}_{\text{emp}})\), where \(\hat{\mu}_{\text{emp}}\) is estimated empirically and, unlike the MV-model inputs, no assumption is made about the data’s underlying distribution.

In the next section we provide a practical illustration of the financial applications of the proposed model in emerging stock market asset allocations. In all MV optimizations carried out we use positive weights and ex post.

### III. Performance of Robust Portfolios: Higher accumulated yields?

We use our estimates \((\hat{\mu}_{\text{emp}}, \hat{\Sigma}_{\text{emp}})\) as inputs for an asset allocation exercise with the MV model to construct robust portfolios that should reflect the behavior of both the normal and the higher volatility days. We stress that they do not reflect extreme crises. We note that portfolios constructed based on high breakdown point estimates are meant to be used for long term objectives, since they capture the dynamics of the majority of the business days.

The seven markets in our emerging market portfolio exercise are: Argentina, Brazil, Mexico, China, Korea, Taiwan, and South Africa. We decided to use a sample of some of the largest emerging markets with representatives of Africa, Asia, and Latin America. We have 2151 daily returns from July 3, 1995 through September 29, 2003. The indices have been obtained from Datastream and are all computed by S&P from the former IFC (International Financial Corporation) Global Indices. The Indices are market capitalization weighted. The market capitalization of the constituents of the S&P/IFCG indices exceeds 75% market value of all domestic shares listed on the local stock exchange. Index computation details may be obtained at Standard and Poor’s website. We used both local currency and dollar returns and computed returns linearly.
III.1 First empirical exercise

We perform out-of-the-sample analysis of several aspects of the optimal robust and classical portfolios. We call “classical” the portfolio that uses the historical sample estimates and “robust” the portfolio that uses the estimates obtained by our contamination technique described in section II. We investigate which portfolio could yield higher cumulative returns. To this end, we split the data in two parts. The first part of the data, the estimating period (the first 1870 daily observations), is used to compute the robust and classical inputs for the MV optimization procedure. The second part, the testing period (281 daily observations), is used in the comparisons. We are interested in the cumulative returns at the end of the testing period. Roughly, the estimation period spans over 6 years and the testing period is the following year. It is common practice to use the previous 5 years to obtain historical estimates, hence our choice of period lengths.

Thus, we analyze the trajectory of the portfolios’ cumulative returns in the testing period. Three portfolios in the efficient frontier were used in the comparisons: (a) the portfolio possessing a fixed target daily percentage return \( v \), say, \( v=0.041\% \); (b) the minimum risk and (c) the maximum return portfolios. Note that even though the robust and the classical portfolios have the same target expected daily return value of 0.041\%, they belong to completely different regions in their respective efficient frontiers. The robust one lies in a low risk region while the classical lies in a high risk region. The reason is that the two efficient frontiers lie on different regions of \( \mathbb{R}^2 \).

The portfolios’ performances are assessed by computing their allocations at baseline \( t = 1870 \) (given in Table 1), which is the end of the estimating period, through \( t = 2151 \), the end of the testing period. The weights were kept fixed during the testing period. The three robust portfolios have lower risk than their classical counterparts. The asset allocation for the robust portfolios is also better distributed among markets and is quite different from the classical portfolio’s weights. Our first empirical exercise indicates that our robust contamination technique yields portfolio weights that dominate the classical portfolio weights in emerging market country allocations, at least for the period examined.

Figure I shows the cumulative returns of the fixed 0.041\% return portfolios. It displays returns accumulated over the 281-day testing period. The portfolio constructed using \( (\mu, \hat{\Sigma}) \) (the black line) dominates the classical one (the gray line). We note that even though the mean return of the robust and the classical portfolios is the same, it seems that the robust method, being more truthful to the data, is more successful when composing the portfolios. We repeated the analysis using local currency returns. The results are qualitatively the same and we do not report them, but make them available upon request, as usual.

III.2 Second empirical exercise

In our first exercise we obtained the portfolio weights at the base day \( t=1870 \) and studied the portfolio behavior for the following 281 days. However, it is possible that the portfolio should be rebalanced more often or that this time horizon is too long. In our second exercise we rebalance the portfolio by computing its covariance matrix and weights every 10 days. The baseline times now are \( t = 1870, 1880, 1890, ..., 2050 \). We have 19 baseline or estimation times. Thus we have 19 baseline portfolios. For each one we compute 19 trajectories and 19 accumulated returns over the 100 days following each baseline.

There are two objectives to this second exercise: (1) to assess the stability of the covariance estimates, as this stability carries over to the weights; (2) to assess and compare accumulated gains over a shorter time horizon of about 4 months (the first exercise assumed
approximately a 1 year horizon). We update the portfolio weights more frequently because economic changes would be captured by the estimates. At each baseline point we enlarge the data set, and add more information. Then we examine the distributions of the returns and risks of the robust and classical portfolios at two points in time: at the baselines (t=1870, 1880, ..., 2050), and also at the end of each of the 100-day periods (t=1971, ..., 2151).

The out-of-the-sample performance of portfolios depends upon their intrinsic characteristics, as well as on whether or not the testing and the estimating periods are compatible. In other words, for the comparisons to be meaningful, the inputs computed with and without the observations in the testing period should be close.

We compute six portfolios at each baseline: the minimum risk (Mi), the maximum return (Ma), and a “central” portfolio (Me), using the classical and the robust covariance matrix. The central portfolio return is the average between the returns of the portfolios of minimum risk and maximum return over its respective efficient frontier. By choosing the “central” portfolio, we aim to characterize a portfolio designed for investors with about the same degree of risk aversion, half way between the minimum and maximum risks for any given efficient frontier. The cumulative return over a 100-day period is computed for each portfolio. Then, the following 10 observations (t = 1871 to t = 1880) are added to the estimating sample. All computations are repeated, robust and classical portfolios of the three types (Mi, Ma, Me) are obtained at the baseline t=1880, and estimates for the final value of the 100-day accumulated returns of all portfolios are saved. We repeat this process until we have 2051 observations in the sample, thus obtaining 19 representations of the returns of the (6) portfolios at the baselines and at the end of each 100-day period.

Figure II shows the distribution of the portfolio returns and risk at the baselines. These are their past returns at the baselines. The notations RMi, RMe, and RMa (CMi, CMe, CMa) stand for the robust (classical) portfolios of the three types. The plot at left shows the returns. We observe that the robust portfolios are more promising, with a distribution located at higher values and possessing less variability. For example, for the minimum variance portfolios, the smaller observed robust value was greater than the highest observed for the classical portfolios. We also carried out a formal paired t-test to verify the equality of the means of returns. For the three portfolio types the p-value was zero against the alternative hypothesis of the robust mean return being greater than the classical. The risks associated to the 19 portfolios are box-plotted at the right hand side of Figure II. We also note a smaller variability of the (also smaller) robust quantities.

All of this was the past. Do the robust portfolios deliver in the testing period? We investigate the distribution of the US dollar returns by examining their 100-day accumulated values associated to the 19 baseline portfolios. Table 2 summarizes our results. We observe that the accumulated returns distributions of the robust portfolios are located to the right of the classical ones for all portfolio types. For example, the central robust portfolio median is 6.37%, whereas the classical central portfolio median is 4.77%. There is a 160 basis point return difference in 100 days, which is highly significant from a financial stand point. The maximum return portfolios are not given but their results are qualitatively similar, albeit less relevant.

We could look to the results by observing the trajectory after the 100 days for each baseline portfolio. We observe the 19 differences between the robust and the classical (100 days) accumulated returns for the minimum variance portfolio with the weights obtained at baselines 1870, 1880, ..., 2050. Thus, for each of the 19 time periods, is the performance of the minimum variance robust portfolios greater? The answer is yes. The boxplots for this comparison, available from the authors, show the differences at the end of each of the 19 trajectories for the minimum variance and the central portfolios. The maximum return
portfolios behave similarly with an allocation of 100% in China. The formal t-tests reject that
the mean difference is equal to zero with zero p-values for both portfolio types.

### III.3 Weight Stability

We now investigate the stability of the robust and classical portfolio weights. This is
important because portfolio holdings may remain fixed for an extended time. We estimate the
weights and rebalance the portfolios daily for 200 days. We form a data set of 200 weights
attached to each emerging market index in our study and examine its concentration, or
stability. Successive days were incorporated into the analysis, one at a time. The sample
started with 1951 observations and increased until it reached 2151 observations. At each of
the 200 baselines we computed the robust and classical MV inputs. The idea is to observe, for
a given portfolio, how weights change as long as new data points are incorporated in the
analysis. The minimum risk, the maximum return, and the “central” portfolios are used.

The boxplot of the weights associated with the 7 components of the minimum risk
portfolio under the robust and classical approaches for the central portfolio, available from the
authors, indicate that the robust weights presented less variability for all 7 components.
However, variables 2 (Brazil), 3 (Korea), and 6 (South Africa) were never used by the
classical procedure and should not be used in the comparisons. The robust portfolios seem to
have more stable weights, thus reducing portfolio rebalancing costs. The weights are very
stable for the minimum risk portfolios, under both robust and classical procedures.

### IV. Conclusions

In this paper we proposed the use of robust inputs for the MV-model. The main
motivations for this work were that a robust estimate should capture the correlations observed
in the vast majority of the business days for long horizon investments with no frequent
updating of portfolio weights; and the fact that extreme observations may show up locally or
globally and may result in spurious correlations if zero breakdown point classical estimates
are used.

Several aspects of the out-of-sample performance of the robust and classical portfolios
were investigated. We found that robust portfolios typically yield higher accumulated returns.
Also, for any given type of portfolio in the efficient frontier, the robust portfolios showed a
more concentrated distribution with higher expected returns. We also concluded that the
baseline choice has a stronger effect on classical portfolios than on the robust ones. In other
words, due to their definition and statistical properties, the robust estimates were able to
reduce instability of the optimization process. Finally, we found that this stability property
carried over to the weights associated with the robust portfolios. We strongly recommend that
a robust covariance matrix be used to solve emerging stock market allocation problems. We
believe that our technique brings a key advantage. Since we only change the covariance
matrix, we can use any commercially available optimizer to obtain robust portfolio weights.

### V. References


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Table 1

Portfolio compositions at baseline t=1870, based on the robust and classical inputs. Daily US dollar returns from S&P/IFCG indexes reported.

<table>
<thead>
<tr>
<th></th>
<th>% Daily Return</th>
<th>% Risk (St. Dev.)</th>
<th>WEIGHTS</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td>ARG</td>
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<tr>
<td>(a) Fixed Target (0.00041) Return Portfolios</td>
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<td></td>
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<tr>
<td>Robust</td>
<td>0.041</td>
<td>0.775</td>
<td>0.085</td>
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<tr>
<td>Classical</td>
<td>0.041</td>
<td>1.539</td>
<td>0.000</td>
</tr>
<tr>
<td>(b) Minimum Risk Portfolios</td>
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<td></td>
<td></td>
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<tr>
<td>Robust</td>
<td>0.033</td>
<td>0.770</td>
<td>0.130</td>
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<tr>
<td>Classical</td>
<td>0.000</td>
<td>1.003</td>
<td>0.088</td>
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<tr>
<td>(c) Maximum Return Portfolios</td>
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<tr>
<td>Robust</td>
<td>0.090</td>
<td>1.419</td>
<td>0.000</td>
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<tr>
<td>Classical</td>
<td>0.043</td>
<td>1.699</td>
<td>0.000</td>
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Table 2

Quantiles of the daily US dollar return distribution of the 100-days cumulative returns of the minimum variance and central portfolios according to the robust and classical estimation procedures.

<table>
<thead>
<tr>
<th>Probabilities</th>
<th>0.05</th>
<th>0.25</th>
<th>0.50</th>
<th>0.75</th>
<th>0.95</th>
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<td>Minimum Risk Portfolios (Mi)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Robust</td>
<td>3.5458</td>
<td>6.0519</td>
<td>10.025</td>
<td>15.3432</td>
<td>20.7501</td>
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<tr>
<td>Classical</td>
<td>0.7757</td>
<td>5.2593</td>
<td>8.5151</td>
<td>14.3825</td>
<td>20.7104</td>
</tr>
<tr>
<td>Central Portfolios (Me)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Robust</td>
<td>-0.3210</td>
<td>4.3651</td>
<td>6.3793</td>
<td>11.9707</td>
<td>18.304</td>
</tr>
<tr>
<td>Classical</td>
<td>-3.7725</td>
<td>1.0911</td>
<td>4.7786</td>
<td>11.8908</td>
<td>17.3109</td>
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</table>
Figure I: Cumulative (%) daily returns of portfolios with target daily return equal to 0.041%.
Figure II: Distribution of the returns (left) and risks (right) of the 19 baseline portfolios \( t = 1870, 1880, \ldots \) for each portfolio type (minimum risk, central, and maximum return) under the robust (R) and classical (C) approaches.